Splitting of Equivalent Points in Noncentrosymmetric Space Groups Into Subsets Under Homogeneous Stress

H. S. Peiser, J. B. Wachtman, Jr., F. A. Munley, and L. C. McCleary

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Splitting of general positions in crystals into subsets of equivalent sites under homogeneous stress has previously been given for all centrosymmetric space groups; the tabulation is here completed for all space groups by listing the results for noncentrosymmetric space groups.

1. Introduction

The present paper presents results analogous to those previously submitted for centrosymmetric space groups [1] ¹ and for point groups [2]; these previous papers should be consulted for detailed discussion.

It is assumed that the symmetry elements possessed by a homogeneously stressed crystal will be those common to the crystal and to the macroscopic state of stress. Application of stress either leaves a space group unaltered or lowers it to a subgroup. Such lowering can always be considered to take place in successive steps each of which leaves no group which is both a stress-induced subgroup of the initial group and a supergroup of the final group, and which is distinct from both. Each such step can be accomplished by a uniaxial stress; for the noncentrosymmetric space groups all but two of the symmetry reductions consisting of two or more successive steps can also be accomplished by uniaxial stress. These two require biaxial stress [2]. More general stress states are, however, consistent with many of the steps of symmetry lowering; we list the most general state of stress (in terms of the modified stress ellipsoid [1]) consistent with each step. The same stress is appropriate for all space-group-to-spacegroup transformations associated with a given pointgroup-to-point-group transformation. There are 25 of the latter which are minimum steps of symmetry lowering for noncentrosymmetric point groups so that the results for the noncentrosymmetric space groups are collected into 25 corresponding tables.

A set of points all of which are equivalent in the unstressed crystal frequently splits into two or more subsets under stress. For each space group all possibilities are taken into account by considering the behavior of the general position because the behavior of each special position can be derived by specializing the general position. This process of

specialization in space groups has been discussed and a technique for visualizing it in terms of stereograms of point groups has been described [2].

2. Results

2.1. Behavior of General Position

The splitting of the general position (set of equivalent sites having no symmetry) into subsets is listed in tables 1 through 25. Each table is headed by a point-group transformation which is a minimum step of symmetry lowering. Each of the space groups associated with the initial point group is listed in the table together with the coordinates of a set of sites making up a general position. The latter are collected into subsets; all of the sites in a subset remain equivalent after symmetry reduction to the final space group which is also listed. For some of the point-group reductions the final point group can occur in two or three non-equivalent orientations. These may correspond to different final space groups; in table 12, for example, one orientation corresponds to the caption at the top of the table and the other to the caption at the bottom. The stress is described by giving conditions on the axes X, Y, Z of the stress ellipsoid [1] to the crystal axes x, y, z; the stress described is the most general (i.e., least restricted) consistent with the symmetry reduction. In many of the tables a single stress specification suffices for all space groups, but in some (table 10, for example) the stress must be specified for each space group because it is customary to choose the axes in different orientations with respect to the point group.

2.2. Stress Table

The most general stress consistent with each possible step of stress-induced symmetry lowering, minimum or compound, is listed in table 26 for all of of the noncentrosymmetric point groups. The stress conditions for the minimum steps of stress-induced symmetry lowering are equivalent, though not always identical, to those given in tables 1–25.

Figures in brackets indicate the literature references on page 462.

Table 1. Reduction from $\frac{1}{4}3m$ to $\frac{1}{4}2m$

of uns	e group strained , order 24 tice point			-				Coe	ordina			to axes				stal							of st crysta	e group rained l, order 8 tice point
No.	Symbol			1	st Subs	et		-			2	2d Subs	et	-			-	3	d Subs	et			No.	Symbol
		(<i>x</i> ,	у,	z) (у,	<i>x</i> ,	z)	(у,	z,	x) (Ζ,	у,	x)	(z,	х,	y) (<i>x</i> ,	z,	<i>y</i>)		
01.5	P43m	(\bar{y} ,	x,	Ē,) (ā,	y,	$\bar{z})$	($\bar{z},$	y,	$\bar{x})$ (\bar{y} ,	z,	$\bar{x})$	(\bar{x} ,	ε,	$\bar{y})$ (ž,	<i>x</i> ,	$\bar{y})$	111	P 4 2m
215	P 43III	(\bar{x} ,	$ar{y}$,	z)(\bar{y} ,	\bar{x} ,	z)	(\bar{y} ,	ž,	x) ($\bar{z},$	$\bar{y},$	x)	(ž,	\bar{x} ,	y) (\bar{x} ,	ž,	y)	111	F 42III
		(y,	\bar{x} ,	$\bar{z})$ (x,	\vec{y} ,	$\bar{z})$	(z,	\bar{y} ,	$\bar{x})$ (y,	ž,	$\bar{x})$	(x,	Ē,	\bar{y}) (z,	\bar{x} ,	$\bar{y})$		
		(x,	у,	z) (у,	<i>x</i> ,	z)	(у,	z,	x) (Ζ,	у,	x)	(z,	<i>x</i> ,	y) (x,	z,	y)		
216	F 4 3m	(\hat{y} ,	x,	$\bar{z})$ (\bar{x} ,	y,	$\bar{z})$	($\bar{z},$	y,	$\bar{x})$ ($\bar{y},$	z,	$\bar{x})$	(\tilde{x} ,	z,	$\bar{y})$ ($\bar{z},$	x,	$\bar{y})$	119	14m2
210	1 40111	(\bar{x} ,	\bar{y} ,	z) (\bar{y} ,	\tilde{x} ,	z)	(\bar{y} ,	ž,	x) ($\bar{z},$	\bar{y} ,	x)	(z,	\bar{x} ,	y) (\bar{x} ,	\bar{z} ,	y)	110	141112
		(<i>y</i> ,	\tilde{x} ,	<u>z</u>)(<i>x</i> ,	\bar{y} ,	Ē)	5	z,	ỹ,	\bar{x}) (у,	z̄,	$\bar{x})$	(x,	ž,	\bar{y}) (z,	\bar{x} ,	$\bar{y})$		
		(<i>x</i> ,	y,	z) (y,	x,	z)	(y,	Ζ,	x) (z,	y,	x)	(z,	x,	y) (x,	z,	y)		
217	143m	(\bar{y} ,	x,	$\bar{z})$ (\bar{x} ,	y,	$\bar{z})$	(ž,	y,	$\bar{x})$ (\bar{y} ,	z,	$\bar{x})$	(\bar{x} ,	z,	$\bar{y})$ (ž,	<i>x</i> ,	$\bar{y})$	121	142m
211	1 10111	(\bar{x} ,	\bar{y} ,	z) (\bar{y} ,	\bar{x} ,	z)	(\bar{y} ,	ž,	x) (z,	\bar{y} ,	x)	(z,	\bar{x} ,	y) (\bar{x} ,	z,	y)	122	
		(<i>y</i> ,	\bar{x} ,	<u>z</u>)(<i>x</i> ,	\bar{y} ,	z̄)	(г,	\bar{y} ,	$\bar{x})$ (у,	ž,	\bar{x})	(x,	ž,	\bar{y}) (z,	z ,	$\bar{y})$	_	
		(х,	y,	z) (½	(2+y, 1)	$2+x, \frac{1}{2}$	(2+z)	(y,	Ζ,	x) (1/2	(z+z, 1)	(2+y, 1)	(2+x)	(<i>z</i> ,	x,	$y) (\frac{1}{2})$	$(2+x, \frac{1}{2})$	(2+z, 1)	(2+y)		
218	P43n	(1/2	(-y, 1)	2+x, 1	(2-z) (\bar{x} ,	y,	ž)	(1/2	$(z-z, \frac{1}{2})$	(2+y, 1)	(2-x) (\bar{y} ,	г,	$\tilde{x})$	(1/2	$(2-x, \frac{1}{2})$	z + z, 1/z	(2-y) (z,	x,	$\bar{y})$	112	P42c
		(\bar{x} ,	\bar{y} ,	z) (½	$(2-y, \frac{1}{2})$	$(2-x, \frac{1}{2})$	(2+z)	(\bar{y} ,	ž,	$x)$ ($\frac{1}{2}$	$(2-z, \frac{1}{2})$	$(2-y, \frac{1}{2})$	(2+x)	(ž,	\bar{x} ,	$y) (\frac{1}{2})$	$(2-x, \frac{1}{2})$	$(2-z, \frac{1}{2})$	(2+y)		
		(1/2	(2+y, 1)	$\sqrt{2}-x, \frac{1}{2}$	(z-z)(<i>x</i> ,	\bar{y} ,	ž)	(1/2	$(2+z, \frac{1}{2})$	(2-y, 1)	<u>(</u> 2−x) (у,	ž,	\bar{x})	(1/2	$(2+x, \frac{1}{2})$	$(2-z, \frac{1}{2})$	(2-y) (z,	\bar{x} ,	\vec{y})	_	
		(<i>x</i> ,	<i>y</i> ,	z) (y,	$x, \frac{1}{2}$		(y,	z,	x) (z,	$y, \frac{1}{2}$	2+x)	(z,	x,	y) (<i>x</i> ,	$z, \frac{1}{2}$	(2+y)		
219	F 43c	(\bar{y} ,	$x, \frac{1}{2}$	(2-z) (\bar{x} ,	y,	ž)	(ž,		(2-x) (\bar{y} ,	z,	\bar{x})	(\bar{x} ,	$z, \frac{1}{2}$	(2-y) (ž,	x,	$\bar{y})$	120	1 4 c2
		(\bar{x} ,	\bar{y} ,	z) (\bar{y} ,	$\bar{x}, \frac{1}{2}$		(\bar{y} ,	z,	x) (ž,		2+x)	(ż,	\bar{x} ,	y) (\bar{x} ,		(2+y)		
		(у,	\bar{x} , $\frac{1}{2}$	½−z)(x,	\bar{y} ,	ž)	(z,	$\bar{y}, \frac{1}{2}$	<u>√</u> 2−x)(у,	ž,	\bar{x})	(x,	<i>z</i> , ½	(2-y) (z,	\bar{x} ,	$\bar{y})$	-	
		(х,	<i>y</i> ,			4 +x, ½		(y,	z,			4+y, ½		(z,	x,		1 +x, ½				
220	1 4 3d				(4-z) ((2+y, 1)					(4-x)(- , ,	2+z, 1				$4+z, \frac{1}{2}$			(2+x, 1)		122	1 4 2d
		.,	<u>´</u> 2− <i>x</i> ,		2+z)(½				"	(2-y,		2+x) (1/2	- ,.		,		½−z,		(2+y) (½					
		(3/2	(+y, 1)	$\sqrt{4}-x, \frac{3}{2}$	$(4-z)(\frac{1}{2})$	$2+x, \frac{1}{2}$	$\sqrt{2}-y$,	ž)	(3/	$1+z, \frac{1}{2}$	$4-y, \frac{3}{2}$	(4-x)(1)	(2+y, 1)	$\sqrt{2}-z$,	\tilde{x})	(3/	$4+x, \frac{1}{2}$	$4-z, \frac{3}{2}$	(-y)(1/2)	$2+z, \frac{1}{2}$	2-x	$\bar{y})$		

3. References

J. B. Wachtman, Jr., and H. S. Peiser, Splitting of a set of equivalent sites in centrosymmetric space groups into subsets under homogeneous stress, J. Res. NBS 69A (Phys. and Chem.) No. 2, 193–207 (1965).
 H. S. Peiser and J. B. Wachtman, Jr., Reduction of crystallographic point groups to subgroups by homogeneous stress, J. Res. NBS69A (Phys. and Chem.) No. 4, 309–324 (1965).

Table 2. Reduction from $\frac{1}{4}3m$ to 3m

unstrain	group of ned crystal, der 24			at $X=Y$; [111] $ Z $		Space group of strained crystal, order 6
per lati	tice point		Coordinates referred to a	xes of unstrained crystal		per lattice point
No.	Symbol	1st Subset	2d Subset	3d Subset	4th Subset	No. Symbol
		(x, y, z)	$($ $x,$ $\bar{y},$ $\bar{z})$	$($ $\bar{x},$ $y,$ $\bar{z})$	$($ $\bar{x},$ $\bar{y},$ $z)$	
		($y,$ $z,$ $x)$	$(\qquad \bar{y}, \qquad \bar{z}, \qquad x)$	$($ $y,$ $\bar{z},$ $\bar{x})$	(\bar{y}, z, \bar{x})	
215	P43m	(z , x , y $)$	$(\qquad \bar{z}, \qquad x, \qquad \bar{y})$	$(\qquad \bar{z}, \qquad \bar{x}, \qquad y)$	$($ $z,$ $\bar{x},$ $\bar{y})$	160 R3m
		($y,$ $x,$ $z)$	$(\qquad \bar{y}, \qquad x, \qquad \bar{z})$	(y, \bar{x}, \bar{z})	$($ $\bar{y},$ $\bar{x},$ $z)$	
		(z, y, x)	$(\qquad \bar{z}, \qquad \bar{y}, \qquad x)$	$($ $\bar{z},$ $y,$ $\bar{x})$	(z, \bar{y}, \bar{x})	
	-	(x, z, y)	$ ($ $x,$ $\bar{z},$ $\bar{y})$	(\bar{x}, \bar{z}, y)	(\bar{x}, z, \bar{y})	
		(x, y, z)	(x, \tilde{y}, \bar{z})	$($ $\bar{x},$ $y,$ $\bar{z})$	$($ $\bar{x},$ $\bar{y},$ $z)$	
		(y, z, x)	$($ $\bar{y},$ $\bar{z},$ $x)$	(y, \bar{z}, \bar{x})	$($ $\bar{y},$ $z,$ $\bar{x})$	
216	F 4 3m	(z, x, y)	$($ $\bar{z},$ $x,$ $\bar{y})$	(\bar{z}, \bar{x}, y)	(z, \bar{x}, \bar{y})	160 R3m
		(y, x, z)	$($ $\bar{y},$ $x,$ $\bar{z})$	(y, \bar{x}, \bar{z})	$($ $\bar{y},$ $\bar{x},$ $z)$	
		(z, y, x)	$($ $\bar{z},$ $\bar{y},$ $x)$	$($ $\bar{z},$ $y,$ $\bar{x})$	(z, \bar{y}, \bar{x})	
	-	(x, z, y)	(x, \bar{z}, \bar{y})	(\bar{x}, \bar{z}, y)	(\bar{x}, z, \bar{y})	
		(x, y, z)	(x, \hat{y}, \bar{z})	$($ $\bar{x},$ $y,$ $\bar{z})$	$($ $\bar{x},$ $\bar{y},$ $z)$	
		(y, z, x)	$($ $\hat{y},$ $\bar{z},$ $x)$	(y, \bar{z}, \bar{x})	(\bar{y}, z, \bar{x})	
217	143m	(z, x, y)	$($ $\bar{z},$ $x,$ $\bar{y})$	$($ $\bar{z},$ $\bar{x},$ $y)$	(z, \bar{x}, \bar{y})	160 R3m
		(y, x, z)	$($ $\bar{y},$ $x,$ $\bar{z})$	(y, \bar{x}, \bar{z})	$($ $\bar{y},$ $\bar{x},$ $z)$	
		(z, y, x)	(\bar{z}, \bar{y}, x)	$($ $\bar{z},$ $y,$ $\bar{x})$	(z, \bar{y}, \bar{x})	
	-	(x, z, y)	(x, \bar{z}, \bar{y})	(\bar{x}, \bar{z}, y)	(\bar{x}, z, \bar{y})	
		(x, y, z)	(x, \bar{y}, \bar{z})	$($ $\bar{x},$ $y,$ $\bar{z})$	$($ $\vec{x},$ $\vec{y},$ $z)$	
		(y, z, x)	$($ $\bar{y},$ $\bar{z},$ $x)$	(y, \bar{z}, \bar{x})	$($ $\bar{y},$ $z,$ $\bar{x})$	
218	P 43n	(z, x, y)	$(\tilde{z}, x, \tilde{y})$	(\bar{z}, \bar{x}, y)	(z, \bar{x}, \bar{y})	161 R3c
		$(\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}+z)$	$(\frac{1}{2} - y, \frac{1}{2} + x, \frac{1}{2} - z)$	$(\frac{1}{2}+y, \frac{1}{2}-x, \frac{1}{2}-z)$	$(\frac{1}{2} - y, \frac{1}{2} - x, \frac{1}{2} + z)$	
		$(\frac{1}{2}+z, \frac{1}{2}+y, \frac{1}{2}+x)$	$(\frac{1}{2}-z, \frac{1}{2}-y, \frac{1}{2}+x)$	$(\frac{1}{2}-z, \frac{1}{2}+y, \frac{1}{2}-x)$	$(\frac{1}{2}+z, \frac{1}{2}-y, \frac{1}{2}-x)$	
	-	$(\frac{1}{2}+x, \frac{1}{2}+z, \frac{1}{2}+y)$	$ \frac{(\frac{1}{2}+x,\frac{1}{2}-z,\frac{1}{2}-y)}{(\frac{1}{2}+x,\frac{1}{2}-z,\frac{1}{2}-y)} $	$(\frac{1}{2}-x,\frac{1}{2}-z,\frac{1}{2}+y)$	$\frac{(\frac{1}{2}-x,\frac{1}{2}+z,\frac{1}{2}-y)}{(\frac{\pi}{2}+$	
		(x, y, z)	(x, \bar{y}, \bar{z})	(\bar{x}, y, \bar{z})	$($ $\bar{x},$ $\bar{y},$ $z)$	
		(y, z, x)	$($ $\bar{y},$ $\bar{z},$ $x)$	(y, \bar{z}, \bar{x})	$($ $\bar{y},$ $z,$ $\bar{x})$	
219	F 43c	(z, x, y)	(\bar{z}, x, \bar{y})	(\bar{z}, \bar{x}, y)	(z, \bar{x}, \bar{y})	161 R3c
		$(y, x, \frac{1}{2} + z)$	$(\bar{y}, x, \frac{1}{2}-z)$ $(\bar{z}, \bar{y}, \frac{1}{2}+x)$	$(y, \bar{x}, \frac{1}{2} - z)$ $(\bar{z}, y, \frac{1}{2} - x)$	$(\bar{y}, \bar{x}, \frac{1}{2} + z)$ $(z, \bar{y}, \frac{1}{2} - x)$	
		$(z, y, \frac{1}{2} + x)$. ,			
	-	$(x, z, \frac{1}{2}+y)$	$(x, \bar{z}, \frac{1}{2} - y)$	$ \begin{array}{ccc} (& \bar{x}, & \bar{z}, \frac{1}{2} + y) \\ \hline & (& \bar{x}, \frac{1}{2} + y, \frac{1}{2} - z) \end{array} $	$\frac{(\bar{x}, z, \frac{1}{2} - y)}{(\frac{1}{2} - x, \bar{y}, \frac{1}{2} + z)}$	
		(x, y, z)	$(\frac{1}{2}+x,\frac{1}{2}-y, \bar{z})$			
		(y, z, x)	$(\frac{1}{2}-y, \bar{z}, \frac{1}{2}+x) $ $(\bar{z}, \frac{1}{2}+x, \frac{1}{2}-y)$	$(\frac{1}{2}+y, \frac{1}{2}-z, \bar{x})$ $(\frac{1}{2}-z, \bar{x}, \frac{1}{2}+y)$	$(\bar{y}, \frac{1}{2} + z, \frac{1}{2} - x)$ $(\frac{1}{2} + z, \frac{1}{2} - x, \bar{y})$	
220	1 4 3d	(z, x, y)		, , , , , , , , , , , , , , , , , , , ,	$(\frac{1}{2}+z, \frac{1}{2}-x, y)$ $(\frac{1}{4}-y, \frac{3}{4}-x, \frac{3}{4}+z)$	161 R3c
		$(\frac{1}{4} + y, \frac{1}{4} + x, \frac{1}{4} + z)$	$(\sqrt[3]{4} - y, \sqrt[3]{4} + x, \sqrt[1]{4} - z)$	$(\sqrt[3]{+}y, \sqrt[1]{4} - x, \sqrt[3]{4} - z)$		
		$(\frac{1}{4}+z, \frac{1}{4}+y, \frac{1}{4}+x)$	$(\frac{1}{4} - z, \frac{3}{4} - y, \frac{3}{4} + x)$	$(\frac{3}{4} - z, \frac{3}{4} + y, \frac{1}{4} - x)$	(34+z, 14-y, 34-x)	
		$(\frac{1}{4}+x, \frac{1}{4}+z, \frac{1}{4}+y)$	$(\sqrt[3]{4}+x,\sqrt[1]{4}-z,\sqrt[3]{4}-y)$	$(\frac{1}{4}-x, \frac{3}{4}-z, \frac{3}{4}+y)$	$(\sqrt[3]{4}-x,\sqrt[3]{4}+z,\sqrt[1]{4}-y)$	

Table 3. Reduction from 432 to 422

of uns	e group strained , order 24		If stressed so that $X{=}Y$; $z\ Z$	Space group of strained crystal, order 8
	tice point	C	ordinates referred to axes of unstrained crysta	per lattice point
No.	Symbol	1st Subset	2d Subset 3d Subset	No. Symbol
207	P432	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	89 P422
208	$\mathrm{P4}_{2}32$	$(x, y, z) (\frac{1}{2} + y, \frac{1}{2} + x, \frac{1}{2} - z)$ $(\frac{1}{2} - y, \frac{1}{2} + x, \frac{1}{2} + z) (\bar{x}, y, \bar{z})$ $(\bar{x}, \bar{y}, z) (\frac{1}{2} - y, \frac{1}{2} - x, \frac{1}{2} - z)$ $(\frac{1}{2} + y, \frac{1}{2} - x, \frac{1}{2} + z) (\bar{x}, \bar{y}, \bar{z})$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	93 P4 ₂ 22
209	F432	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	97 1422
210	$\mathrm{F4_{l}32}$	$(x, y, z)(4+y, 1/4+x, 1/4-z)$ $(1/4-y, 1/4+x, 1/4+z)(\bar{x}, y, \bar{z})$ $(\bar{x}, \bar{y}, z)(1/4-y, 1/4-x, 1/4-z)$ $(1/4+y, 1/4-x, 1/4+z)(x, \bar{y}, \bar{z})$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	98 I4 ₁ 22
211	I432	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	97 I422
212	P4332	$(x, y, z) (\cancel{1}4+y, \cancel{3}4+x, \cancel{3}4-z)$ $(\cancel{3}4-y, \cancel{1}4+x, \cancel{3}4+z) (\overline{x}, \cancel{1}2+y, \cancel{1}2-z)$ $(\cancel{1}2-x, \overline{y}, \cancel{1}2+z) (\cancel{1}4-y, \cancel{1}4-x, \cancel{1}4-z)$ $(\cancel{3}4+y, \cancel{3}4-x, \cancel{1}4+z) (\cancel{1}2+x, \cancel{1}2-y, \overline{z})$		96 P4 ₃ 2 ₁ 2
213	P4 ₁ 32	$ (x, y, z) (34+y, 34+x, 14-z) $ $ (14-y, 34+x, 14+z) (\bar{x}, 1/2+y, 1/2-z) $ $ (1/2-x, \bar{y}, 1/2+z) (34-y, 34-x, 34-z) $ $ (1/4+y, 1/4-x, 3/4+z) (1/2+x, 1/2-y, \bar{z}) $	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	92 P4 ₁ 2 ₁ 2
214	I4 ₁ 32	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	98 I4 ₁ 22

Table 4. Reduction from 432 to 32

Space	group of led crystal -		If stressed so th	aat $X = Y$; [111] Z			group of d crystal
ore	der 24 tice point		Coordinates referred to a	exes of unstrained crystal		ord	ler 6 ice point
No.	Symbol	1st Subset	2d Subset	3d Subset	4th Subset	No.	Symbo
207	P432	($ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	155	R32
208	P4 ₂ 32	(x, y, z) (y, z, x) (z, x, y) $(\frac{1}{2}y, \frac{1}{2}x, \frac{1}{2}z)$ $(\frac{1}{2}z, \frac{1}{2}y, \frac{1}{2}z)$ $(\frac{1}{2}z, \frac{1}{2}z, \frac{1}{2}z)$ $(\frac{1}{2}z, \frac{1}{2}z, \frac{1}{2}z)$	$(x, \ \bar{y}, \ \bar{z})$ $(\bar{y}, \ \bar{z}, \ x)$ $(\bar{z}, \ x, \ \bar{y})$ $(\cancel{2}+y, \cancel{2}-x, \cancel{1}+z)$ $(\cancel{2}+z, \cancel{2}+y, \cancel{2}-x)$ $(\cancel{2}-x, \cancel{2}+z, \cancel{2}+y)$	$(\begin{tabular}{cccccccccccccccccccccccccccccccccccc$	(\bar{x}, \bar{y}, z) (\bar{y}, z, \bar{x}) (z, \bar{x}, \bar{y}) $(\cancel{2} + y, \cancel{1} \cancel{2} + z, \cancel{1} \cancel{2} - z)$ $(\cancel{1} \cancel{2} - z, \cancel{1} \cancel{2} + y, \cancel{1} \cancel{2} + x)$ $(\cancel{1} \cancel{2} + x, \cancel{1} \cancel{2} - z, \cancel{1} \cancel{2} + y)$	155	R32
209	F432	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	155	R32
210	F4 ₁ 32	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	155	R32
211	I432		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	155	R32
212	P4 ₃ 32	$ (x, y, z) $ $ (y, z, x) $ $ (z, x, y) $ $ (\cancel{1}_4 - y, \cancel{1}_4 - x, \cancel{1}_4 - z) $ $ (\cancel{1}_4 - z, \cancel{1}_4 - y, \cancel{1}_4 - x) $ $ (\cancel{1}_4 - x, \cancel{1}_4 - z, \cancel{1}_4 - y) $	$(\frac{1}{2}+x,\frac{1}{2}-y, \bar{z})$ $(\frac{1}{2}-y, \bar{z},\frac{1}{2}+x)$ $(\bar{z},\frac{1}{2}+x,\frac{1}{2}-y)$ $(\frac{3}{4}+y,\frac{3}{4}-x,\frac{1}{4}+z)$ $(\frac{1}{4}+z,\frac{3}{4}+y,\frac{3}{4}-x)$ $(\frac{3}{4}-x,\frac{1}{4}+z,\frac{3}{4}+y)$	$(\bar{x}, \sqrt{2}+y, \sqrt{2}-z)$ $(\sqrt{2}+y, \sqrt{2}-z, \bar{x})$ $(\sqrt{2}-z, \bar{x}, \sqrt{2}+y)$ $(\sqrt{3}4-y, \sqrt{4}+x, \sqrt{3}4+z)$ $(\sqrt{4}+z, \sqrt{3}4-y, \sqrt{4}+x)$ $(\sqrt{4}+x, \sqrt{4}+z, \sqrt{4}-y)$	$(\frac{1}{2}-x, \overline{y}, \frac{1}{2}+z)$ $(\overline{y}, \frac{1}{2}+z, \frac{1}{2}-x)$ $(\frac{1}{2}+z, \frac{1}{2}-x, \overline{y})$ $(\frac{1}{4}+y, \frac{3}{4}+x, \frac{3}{4}-z)$ $(\frac{3}{4}-z, \frac{1}{4}+y, \frac{3}{4}+x)$ $(\frac{3}{4}+x, \frac{3}{4}-z, \frac{1}{4}+y)$	155	R32
213	P4 ₁ 32	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$(\frac{1}{2} + x, \frac{1}{2} - y, \frac{1}{2})$ $(\frac{1}{2} - y, \frac{1}{2}, \frac{1}{2} + x)$ $(\frac{1}{2} + y, \frac{1}{2} - y)$ $(\frac{1}{4} + y, \frac{1}{4} - x, \frac{3}{4} + z)$ $(\frac{3}{4} + z, \frac{1}{4} + y, \frac{1}{4} - x)$ $(\frac{1}{4} - x, \frac{3}{4} + z, \frac{1}{4} + y)$	$(\bar{x}, \frac{1}{2} + y, \frac{1}{2} - z)$ $(\frac{1}{2} + y, \frac{1}{2} - z, \bar{x})$ $(\frac{1}{2} - z, \bar{x}, \frac{1}{2} + y)$ $(\frac{1}{4} - y, \frac{3}{4} + x, \frac{1}{4} + z)$ $(\frac{1}{4} + z, \frac{1}{4} - y, \frac{3}{4} + x)$ $(\frac{3}{4} + x, \frac{1}{4} + z, \frac{1}{4} - y)$	$(\frac{1}{2}-x, \frac{1}{2}, \frac{1}{2}+z)$ $(\frac{1}{2}-x, \frac{1}{2}+z, \frac{1}{2}-x)$ $(\frac{1}{2}+z, \frac{1}{2}-x, \frac{1}{2})$ $(\frac{1}{2}+z, \frac{1}{2}-x, \frac{1}{2}+x)$ $(\frac{1}{2}+z, \frac{1}{2}+x, \frac{1}{2}+x)$ $(\frac{1}{2}+z, \frac{1}{2}+x)$ $(\frac{1}{2}+x, \frac{1}{2}+z, \frac{1}{2}+x)$	155	K32
214	I4 ₁ 32	(x, y, z) (y, z, x) (z, x, y) $(\cancel{1}4-y, \cancel{1}4-x, \cancel{1}4-z)$ $(\cancel{1}4-z, \cancel{1}4-y, \cancel{1}4-x)$ $(\cancel{1}4-x, \cancel{1}4-z, \cancel{1}4-y)$	$(\frac{1}{2}+x,\frac{1}{2}-y, \bar{z})$ $(\frac{1}{2}-y, \bar{z},\frac{1}{2}+x)$ $(\bar{z},\frac{1}{2}+x,\frac{1}{2}-y)$ $(\frac{3}{4}+y,\frac{3}{4}-x,\frac{1}{4}+z)$ $(\frac{1}{4}+z,\frac{3}{4}+y,\frac{3}{4}-x)$ $(\frac{3}{4}-x,\frac{1}{4}+z,\frac{3}{4}+y)$	$(\bar{x}, \frac{1}{2} + y, \frac{1}{2} - z)$ $(\frac{1}{2} + y, \frac{1}{2} - z, \bar{x})$ $(\frac{1}{2} - z, \bar{x}, \frac{1}{2} + y)$ $(\frac{3}{4} - y, \frac{1}{4} + x, \frac{3}{4} + z)$ $(\frac{3}{4} + z, \frac{3}{4} - y, \frac{1}{4} + x)$ $(\frac{1}{4} + x, \frac{3}{4} + z, \frac{3}{4} - y)$	$(\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z)$ $(\bar{y}, \frac{1}{2}+z, \frac{1}{2}-x)$ $(\frac{1}{2}+z, \frac{1}{2}-x, \bar{y})$ $(\frac{1}{4}+y, \frac{3}{4}+x, \frac{3}{4}-z)$ $(\frac{3}{4}-z, \frac{1}{4}+y, \frac{3}{4}+x)$ $(\frac{3}{4}+x, \frac{3}{4}-z, \frac{1}{4}+y)$	155	R32

Table 5. Reduction from 23 to 222

crystal.	e group strained , order 12 cice point			ressed so ———— ordinate									of st	e group rained al, order ttice point
No.	Symbol	1st 8	Subset			2d S	ubset			3d S	ubset		No.	Symbo
		(x,	у,	<i>z</i>)	(y,	z,	<i>x</i>)	(z,	<i>x</i> ,	y)		
195	P23	(x,	\bar{y} ,	$\bar{z})$	(y,	$\bar{z},$	$\bar{x})$	(Ζ,	\bar{x} ,	$\bar{y})$	16	P222
190	1 20	(\bar{x} ,	y,	\bar{z})	(\bar{y} ,	z,	$\bar{x})$	($\bar{z},$	x,	$\bar{y})$	10	1 222
		$(\bar{x},$	\bar{y} ,	<i>z</i>)	(\bar{y} ,	ž,	x)	(ž,	\bar{x} ,	<i>y</i>)		
		(<i>x</i> ,	y,	<i>z</i>)	(y,	z,	x)	(Ζ,	x,	y)		
196	F23	(<i>x</i> ,	\bar{y} ,	$\bar{z})$	(y,	$\bar{z},$	$\bar{x})$	(Ζ,	\bar{x} ,	$\bar{y})$	22	F222
100	120	$(\bar{x},$	y,	$\bar{z})$	(\bar{y} ,	<i>z</i> ,	$\bar{x})$	(ž,	x,	$\bar{y})$		
		$(\bar{x},$	\bar{y} ,	<i>z</i>)	(\bar{y} ,	ž,	<i>x</i>)	(ž,	\bar{x} ,	<i>y</i>)	_	
		(<i>x</i> ,	y,	<i>z</i>)	(y,	z,	x)	(Ζ,	x,	y)		
197	123	(<i>x</i> ,	\bar{y} ,	$\bar{z})$	(y,	z,	\bar{x})	(Ζ,	\bar{x} ,	$\bar{y})$	23	I222
		$(\bar{x},$	y,	$\bar{z})$	(\bar{y} ,	z	\bar{x})	(ž,	x,	$\bar{y})$		
		(<i>x</i> ,	\bar{y} ,	z)	(\bar{y} ,	ž,	x)	(ž,	\bar{x} ,	<i>y</i>)		
		(x,	y,	<i>z</i>)	(y,	z,	x)	(z,	x,	y)		
198	P2 ₁ 3	$(\frac{1}{2}+x,\frac{1}{2}$		Ž)	(1/2	$+y$, $\frac{1}{2}$		\bar{x})	(1/2	$+z, \frac{1}{2}$		$\bar{y})$	19	$P2_{1}2_{1}2$
			2+y, ½		. (2+z, ½		($x + x, \frac{1}{2}$			
	-	$(\frac{1}{2}-x,$					$\bar{z}, \frac{1}{2}$		(1/2		$\bar{x}, \frac{1}{2}$			
		(x,			(0,		<i>x</i>)	(z,		<i>y</i>)		
199	I2 ₁ 3	(½+x, ½			(1/2		<u>(</u> −z,				-x,		24	$I2_{1}2_{1}2_{1}$
			2+y, ½		($z+z, \frac{1}{2}$				$+x, \frac{1}{2}$			1212121
		$(\frac{1}{2}-x,$	$\bar{y}, \frac{1}{2}$	(2+z)	$(\frac{1}{2})$	-y,	$\bar{z}, \frac{1}{2}$	(2+x)	(1/2	− z,	$\bar{x}, \frac{1}{2}$	+y)		

Table 6. Reduction from 23 to 3

unstraine	group of ed crystal,		If s	tressed	so that $X=Y$; [111] $ Z $		strained	group of l crystal,
	12 per e point		Coordinate	s referr	ed to axes of unstrained	erystal		e point
No.	Symbol	1st Subset	2d Subset		3d Subset	4th Subset	No.	Symbol
		(x, y, z)	$(x, \bar{y},$	\bar{z})	$($ $\bar{x},$ $y,$ $\bar{z})$	$($ \bar{x} , \bar{y} , z $)$		
195	P23	(y, z, x)	$(\qquad \bar{y},\qquad \bar{z},$	x)	$($ $y,$ $\bar{z},$ $\bar{x})$	$(\qquad \bar{y}, \qquad z, \qquad \bar{x})$	146	R3
		(z, x, y)	$(\qquad \bar{z}\qquad x,$	$\bar{y})$	$($ $\bar{z},$ $\bar{x},$ $y)$	$($ $z,$ $\bar{x},$ $\bar{y})$		
		(x, y, z)	$(x, \bar{y},$	$\bar{z})$	$($ $\bar{x},$ $y,$ $\bar{z})$	$($ $\bar{x},$ $\bar{y},$ $z)$		
196	F23	(y, z, x)	$(\qquad \bar{y}, \qquad \bar{z},$	x)	$($ $y,$ $\bar{z},$ $\bar{x})$	$(\qquad ilde{y}, \qquad z, \qquad ilde{x})$	146	R3
		(z, x, y)	$(\qquad \bar{z}, \qquad x,$	$\bar{y})$	$($ $\bar{z},$ $\bar{x},$ $y)$	$($ $z,$ $\bar{x},$ $\bar{y})$		
		(x, y, z)	$(x, \bar{y},$	z)	$($ $\bar{x},$ $y,$ $\bar{z})$	$($ $\bar{x},$ $\bar{y},$ $z)$		
197	I23	(y, z, x)	$(\qquad \bar{y},\qquad \bar{z},$	x)	$($ $y,$ $\bar{z},$ $\bar{x})$	$($ \bar{y} , z , \bar{x} $)$	146	R3
	· A1	(z, x, y)	$($ $\bar{z},$ $x,$	$\hat{y})$	$($ $\bar{z},$ $\bar{x},$ $y)$	$($ $z,$ $\bar{x},$ $\bar{y})$		
		(x, y, z)	$(\frac{1}{2}+x,\frac{1}{2}-y,$	z)	$(\bar{x}, \frac{1}{2} + y, \frac{1}{2} - z)$	$(\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z)$		
198	P2 ₁ 3	(y, z, x)	$(\frac{1}{2}-y, \bar{z}, \frac{1}{2}-y)$	+ x)	$(\frac{1}{2}+y,\frac{1}{2}-z, \bar{x})$	$(\bar{y}, \frac{1}{2} + z, \frac{1}{2} - x)$	146	R3
		(z, x, y)	$(\bar{z}, \frac{1}{2} + x, \frac{1}{2} -$	-y)	$(\frac{1}{2}-z, \bar{x}, \frac{1}{2}+y)$	$(\frac{1}{2}+z,\frac{1}{2}-x, \bar{y})$		
		(x, y, z)	$(\frac{1}{2}+x,\frac{1}{2}-y,$	ž)	$(\bar{x}, \frac{1}{2} + y, \frac{1}{2} - z)$	$(\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z)$		
199	I2 ₁ 3	(y, z, x)	$(\frac{1}{2}-y, \bar{z}, \frac{1}{2}-$	[-x)	$(\frac{1}{2}+y,\frac{1}{2}-z, \bar{x})$	$($ $\bar{y}, \frac{1}{2} + z, \frac{1}{2} - x)$	146	R3
		(z,x,y)	$(\tilde{z}, \frac{1}{2} + x, \frac{1}{2} -$	-y)	$(\frac{1}{2}-z, \bar{x}, \frac{1}{2}+y)$	$(\frac{1}{2}+z,\frac{1}{2}-x, \hat{y})$		

Table 7. Reduction from $\bar{6}m2$ to mm2

ınstrain order 12	group of ed crystal, per lattice oint		at y and $z \parallel$ any two		Space group of strained crystal order 4 per latti point		
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbo	
187	Pēm2	(x, y, z) (x, x-y, z) (x, y, \bar{z}) $(x, x-y, \bar{z})$	$ \begin{array}{cccc} (\bar{y},x-y,&z) \\ (\bar{y},&\bar{x},&z) \\ (\bar{y},x-y,&\bar{z}) \\ (\bar{y},&\bar{x},&\bar{z}) \end{array} $		38	Amm2	
188	P 6c2	(x, y, z) $(x, x-y, \frac{1}{2}+z)$ $(x, y, \frac{1}{2}-z)$ $(x, x-y, \bar{z})$	$(\bar{y}, x-y, z)$ $(\bar{y}, \bar{x}, 1/2+z)$ $(\bar{y}, x-y, 1/2-z)$ $(\bar{y}, \bar{x}, \bar{z})$	$(y-x, \bar{x}, z)$ $(y-x, y, \frac{1}{2}+z)$ $(y-x, \bar{x}, \frac{1}{2}-z)$ $(y-x, y, \bar{z})$	40	Ama2	
189	P ¯ 2m	(x, y, z) $(\bar{x}, y-x, z)$ (x, y, \bar{z}) $(\bar{x}, y-x, \bar{z})$		$(y-x, \bar{x}, z)$ $(x-y, \bar{y}, z)$ $(y-x, \bar{x}, \bar{z})$ $(x-y, \bar{y}, \bar{z})$	38	Amm	
190	P ¯ 62e	(x, y, z) $(\bar{x}, y-x, \frac{1}{2}+z)$ $(x, y, \frac{1}{2}-z)$ $(\bar{x}, y-x, \bar{z})$	$(\bar{y}, x-y, z)$ $(y, x, \frac{1}{2}+z)$ $(\bar{y}, x-y, \frac{1}{2}-z)$ (y, x, \bar{z})	$(y-x, \bar{x}, z)$ $(x-y, \bar{y}, \frac{1}{2}+z)$ $(y-x, \bar{x}, \frac{1}{2}-z)$ $(x-y, \bar{y}, \bar{z})$	40	Ama2	

Table 8. Reduction from 6mm to mm2

unstrain	group of ed crystal, per lattice	If stressed so th	at y and $z \parallel$ any two	of X , Y , and Z	strained	group of crystal,
	oint	Coordinates re	eferred to axes of uns	trained crystal		per lattice pint
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbol
183	P6mm	(x, y, z) $(x, x-y, z)$ (\bar{x}, \bar{y}, z) $(\bar{x}, y-x, z)$	$(\bar{y}, x-y, z)$ (\bar{y}, \bar{x}, z) $(y, y-x, z)$ (y, x, z)	$(y-x, \bar{x}, z)$ (y-x, y, z) (x-y, x, z) $(x-y, \bar{y}, z)$	35	Cmm2
184	P6cc	(x, y, z) $(x, x-y, \frac{1}{2}+z)$ (\bar{x}, \bar{y}, z) $(\bar{x}, y-x, \frac{1}{2}+z)$	$(\bar{y}, x-y, z)$ $(\bar{y}, \bar{x}, \frac{1}{2}+z)$ $(y, y-x, z)$ $(y, x, \frac{1}{2}+z)$	$(y-x, \bar{x}, z)$ $(y-x, y, \frac{1}{2}+z)$ $(x-y, x, z)$ $(x-y, \bar{y}, \frac{1}{2}+z)$	37	Ccc2
185	P63cm	(x, y, z) $(x, x-y, \frac{1}{2}+z)$ $(\bar{x}, \bar{y}, \frac{1}{2}+z)$ $(\bar{x}, y-x, z)$	$(\bar{y}, x-y, z)$ $(\bar{y}, \bar{x}, 1/2+z)$ $(y, y-x, 1/2+z)$ (y, x, z)	$(y-x, \bar{x}, z)$ $(y-x, y, \frac{1}{2}+z)$ $(x-y, x, \frac{1}{2}+z)$ $(x-y, \bar{y}, z)$	36	Cmc2 ₁
186	P63mc	(x, y, z) $(x, x-y, z)$ $(\bar{x}, \bar{y}, \frac{1}{2}+z)$ $(\bar{x}, y-x, \frac{1}{2}+z)$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$(y-x, \bar{x}, z)$ $(y-x, y, z)$ $(x-y, x, \frac{1}{2}+z)$ $(x-y, \bar{y}, \frac{1}{2}+z)$	36	Cme2 ₁

Table 9. Reduction from 622 to 222

unstrain order 12	group of ed crystal, per lattice oint		$\begin{array}{c} \text{nat } y \text{ and } z \parallel \text{any two} \\ \\ \text{eferred to axes of uns} \end{array}$		straine order 4	group of d crystal, per lattice oint
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbo
		(x, y, z)	$(\bar{y}, x-y, z)$	$(y-x, \bar{x}, z)$		
	Dana	(x, x-y,	$(\bar{y}, \bar{x}, \bar{z})$	$(y-x,y, \bar{z})$		
177	P622	(\bar{x}, \bar{y}, z)	(y, y-x, z)	(x-y, x, z)	24	
		$(\bar{x}, y-x, \bar{z})$	(y, x, \bar{z})	$(x-y, \bar{y}, \bar{z})$	21	C222
		(x, y, z)	$(\bar{y}, x-y, \frac{1}{3}+z)$	$(y-x, \tilde{x}, \frac{2}{3}+z)$	-	
150	Tra 00	$(x, x-y, \frac{1}{6}-z)$	$(\bar{y}, \bar{x}, 56-z)$	$(y-x, y, \frac{1}{2}-z)$	20	Gaaa
178	P6 ₁ 22	$(\bar{x}, \tilde{y}, \frac{1}{2} + z)$	(y, y-x, 5/6+z)	$(x-y, x, \frac{1}{2}6+z)$	20	C222 ₁
		$(\bar{x}, y-x, \frac{2}{3}-z)$	$(y, x, \frac{1}{3} - z)$	$(x-y, \bar{y}, \qquad \bar{z})$		
		(x, y, z)	$(\bar{y}, x-y, \frac{2}{3}+z)$	$(y-x, \bar{x}, \frac{1}{3}+z)$		
179	P6522	$(x, x-y, \frac{5}{6}-z)$	$(\bar{y}, \bar{x}, \frac{1}{2}6-z)$	$(y-x, y, \frac{1}{2}-z)$	20	C2221
179	F 0522	$(\bar{x}, \bar{y}, \frac{1}{2} + z)$	$(y, y-x, \frac{1}{2}6+z)$	(x-y, x, 56+z)	20	C2221
		$(\bar{x}, y-x, \frac{1}{3}-z)$	$(y, x, \frac{2}{3}-z)$	$(x-y, \bar{y}, \bar{z})$		
		(x, y, z)	$(\bar{y}, x-y, \frac{2}{3}+z)$	$(y-x, \bar{x}, \frac{1}{3}+z)$		
180	P6 ₂ 22	$(x, x-y, \frac{1}{3}-z)$	$(\bar{y}, \bar{x}, \frac{2}{3} - z)$	$(y-x,y, \bar{z})$	21	C222
100	1 0222	(\bar{x}, \bar{y}, z)	$(y, y-x, \frac{2}{3}+z)$	$(x-y, x, \frac{1}{3}+z)$	21	0222
		$(\bar{x}, y-x, \frac{1}{3}-z)$	$(y, x, \frac{2}{3} - z)$	$(x-y, \bar{y}, \bar{z})$		
		(x, y, z)	$(\vec{y}, x-y, \frac{1}{3}+z)$	$(y-x, \bar{x}, \frac{2}{3}+z)$		
181	P6422	$(x, x-y, \frac{2}{3}-z)$	$(\bar{y}, \bar{x}, \frac{1}{3} - z)$	$(y-x,y, \bar{z})$	21	C222
101	1 0422	(\bar{x}, \bar{y}, z)	$(y, y-x, \frac{1}{3}+z)$	$(x-y, x, \frac{2}{3}+z)$	21	0222
		$(\bar{x}, y-x, \frac{2}{3}-z)$	$(y, x, \frac{1}{3} - z)$	$(x-y, \bar{y}, \bar{z})$		
		(x, y, z)	$(\bar{y}, x - y, z)$	$(y-x,\bar{x}, z)$		
182	P6322	$(x, x-y, \frac{1}{2}-z)$	$(\bar{y}, \bar{x}, \frac{1}{2}-z)$	$(y-x, y, \frac{1}{2}-z)$	20	C2221
102	1.0322	$(\bar{x}, \bar{y}, \frac{1}{2} + z)$	$(y, y-x, \frac{1}{2}+z)$	$(x-y, x, \frac{1}{2}+z)$	20	02221
		$(\bar{x}, y-x, \qquad \bar{z})$	(y, x, \bar{z})	$(x-y, \bar{y}, \bar{z})$		

Table 10. Reduction from $4\overline{2}m$ to mm2

unstrain order 8	group of ed crystal, per lattice pint	If stressed so that			Space group of strained crystal, order 4 per lattic point			
No.	Symbol			1st Subset	2d Subset		No.	Symbol
111	P42m	[1 $\overline{10}$], [110], $z \parallel X, Y, Z$, any permutation.	(x, y, z) (\bar{x}, \bar{y}, z)	$($ $y,$ $x,$ $z)$ $($ $\bar{y},$ $\bar{x},$ $z)$	$($ $x,$ $\bar{y},$ $\bar{z})$ $($ $\bar{x},$ $y,$ $\bar{z})$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	35	Cmm2
112	P42e	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(y, x, \frac{1}{2}+z)$ $(\bar{y}, \bar{x}, \frac{1}{2}+z)$	$(x, \bar{y}, \frac{1}{2}-z)$ $(\bar{x}, y, \frac{1}{2}-z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	37	Ccc2
113	P421m	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2} + y, \frac{1}{2} + x, z)$ $(\frac{1}{2} - y, \frac{1}{2} - x, z)$	$(\frac{1}{2}+x, \frac{1}{2}-y, \bar{z})$ $(\frac{1}{2}-x, \frac{1}{2}+y, \bar{z})$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	35	Cmm2
114	P421c	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}+z)$ $(\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}+z)$	$(\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}-z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	37	Ccc2
115	P4m2	$x, y, z \parallel X, Y, Z, $ any permutation.	(x, y, z) (\bar{x}, \bar{y}, z)	(x,	(y, x, \bar{z}) $(\bar{y}, \bar{x}, \bar{z})$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	25	Pmm
116	P4c2	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	(x,	$(y, x, \frac{1}{2}-z)$ $(\bar{y}, \bar{x}, \frac{1}{2}-z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	27	Pec2
117	P4b2	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+x, \frac{1}{2}-y, z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, z)$	$(\frac{1}{2} + y, \frac{1}{2} + x, \qquad \bar{z})$ $(\frac{1}{2} - y, \frac{1}{2} - x, \qquad \bar{z})$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	32	Pba2
118	P4n2	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z)$	$(\frac{1}{2} + y, \frac{1}{2} + x, \frac{1}{2} - z)$ $(\frac{1}{2} - y, \frac{1}{2} - x, \frac{1}{2} - z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	34	Pnn2
119	$1\overline{4}$ m2	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	(x, \bar{y}, z) (\bar{x}, y, z)		(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	44	Imm2
120	14c2	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(x, \bar{y}, \frac{1}{2}+z)$ $(\bar{x}, y, \frac{1}{2}+z)$	$(y, x, \frac{1}{2}-z)$ $(\bar{y}, \bar{x}, \frac{1}{2}-z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	45	Iba2
121	I 4 2m	[1 $\bar{1}$ 0], [110], $z \parallel X$, Y , Z , any permutation.	(x, y, z) (\bar{x}, \bar{y}, z)	(y, x, z) (\bar{y}, \bar{x}, z)	(x,	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	42	Fmm2
122	I 4 2d	Do.	(x, y, z) (x, y, z) (x, \bar{y}, z)	$(y, \frac{1}{2} + x, \frac{1}{4} + z)$ $(\bar{y}, \frac{1}{2} - x, \frac{1}{4} + z)$	$(x, \frac{1}{2} - y, \frac{1}{4} - z)$ $(\bar{x}, \frac{1}{2} + y, \frac{1}{4} - z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	43	Fdd2

Table 11. Reduction from $4\overline{2}m$ to 222

unstrain order 8	group of ed crystal, per lattice pint	If stressed so that		Coordinates referred to a	xes of unstrained crystal		Space group of strained crystal, order 4 per lattice point	
No.	Symbol			1st Subset	2d Subset		No.	Symbol
111	P42m	$x, y, z \parallel X, Y, Z,$ any permutation.	(x, y, z) (\bar{x}, \bar{y}, z)	(x, \bar{y}, \bar{z}) (\bar{x}, y, \bar{z})	(y, x, z) (\bar{y}, \bar{x}, z)	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	16	P222
112	P 42e	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(x, \bar{y}, \frac{1}{2}-z)$ $(\bar{x}, y, \frac{1}{2}-z)$	$(y, x, \frac{1}{2}+z)$ $(\bar{y}, \bar{x}, \frac{1}{2}+z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	16	P222
113	P421m	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+x, \frac{1}{2}-y, \bar{z})$ $(\frac{1}{2}-x, \frac{1}{2}+y, \bar{z})$	$(\frac{1}{2}+y, \frac{1}{2}+x, z)$ $(\frac{1}{2}-y, \frac{1}{2}-x, z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	18	P2 ₁ 2 ₁ 2
114	P421c	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}-z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z)$	$(\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}+z)$ $(\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}+z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	18	P2 ₁ 2 ₁ 2
115	P 4 m2	[1 $\overline{10}$], [110], $z \parallel X$, Y , Z , any permutation.	(x, y, z) (\bar{x}, \bar{y}, z)	(y, x, \bar{z}) $(\bar{y}, \bar{x}, \bar{z})$	(x, \bar{y}, z) (\bar{x}, y, z)	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	21	C222
116	P4c2	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(y, x, \frac{1}{2} - z)$ $(\bar{y}, \bar{x}, \frac{1}{2} - z)$	(x,	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	21	C222
117	P4b2	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+y, \frac{1}{2}+x, \bar{z})$ $(\frac{1}{2}-y, \frac{1}{2}-x, \bar{z})$	$(\frac{1}{2}+x,\frac{1}{2}-y, z)$ $(\frac{1}{2}-x,\frac{1}{2}+y, z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	21	C222
118	P4n2	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}-z)$ $(\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}-z)$	$(\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	21	C222
119	I 4̄m2	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	(y, x, \bar{z}) $(\bar{y}, \bar{x}, \bar{z})$	(x,	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	22	F222
120	14c2	Do.	(x, y, z) (\bar{x}, \bar{y}, z)		$(x, \bar{y}, \frac{1}{2}+z)$ $(\bar{x}, y, \frac{1}{2}+z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	22	F222
121	1 4 2m	$x, y, z \parallel X, Y, Z,$ any permutation.	(x, y, z) (\bar{x}, \bar{y}, z)	(x,	(y, x, z) (\bar{y}, \bar{x}, z)	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	23	I222
122	1 4 2d	Do.	(x, y, z) (\bar{x}, \bar{y}, z)	$(x, \frac{1}{2} - y, \frac{1}{4} - z)$ $(\bar{x}, \frac{1}{2} + y, \frac{1}{4} - z)$	$(y, \frac{1}{2} + x, \frac{1}{4} + z)$ $(\bar{y}, \frac{1}{2} - x, \frac{1}{4} + z)$	(y, \bar{x}, \bar{z}) (\bar{y}, x, \bar{z})	24	I2 ₁ 2 ₁ 2 ₁

Table 12. Reduction from 4mm to mm2

unstrain order 8	group of ned crystal, per lattice oint			$y, z \mid\mid X, Y, Z$, any permud to axes of unstrained cry		Space gr strained order 4 pe	erystal, er lattice		
No.	Symbol		1st Subset	2d Su	bset	No.	Symbol		
99	P4mm	(x, y, z) (\bar{x}, \bar{y}, z)	(x, \bar{y}, z) (\bar{x}, y, z)	(y, x, z) (\bar{y}, \bar{x}, z)	(y, \bar{x}, z) (\bar{y}, x, z)	25	Pmm2	35	Cmm2
100	P4bm	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+x,\frac{1}{2}-y, z)$ $(\frac{1}{2}-x,\frac{1}{2}+y, z)$	$(\frac{1}{2}+y, \frac{1}{2}+x, z)$ $(\frac{1}{2}-y, \frac{1}{2}-x, z)$	(y, \bar{x}, z) (\bar{y}, x, z)	32	Pba2	35	Cmm
101	P4 ₂ cm	(x, y, z) (\bar{x}, \bar{y}, z)	$(x, \bar{y}, \frac{1}{2}+z)$ $(\bar{x}, y, \frac{1}{2}+z)$	(y, x, z) (\bar{y}, \bar{x}, z)	$(y, \bar{x}, \frac{1}{2}+z)$ $(\bar{y}, x, \frac{1}{2}+z)$	27	Pee2	35	Cmm
102	$\mathrm{P4}_{2}\mathrm{nm}$	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z)$	(y, x, z) (\bar{y}, \bar{x}, z)	$(\frac{1}{2}+y,\frac{1}{2}-x,\frac{1}{2}+z)$ $(\frac{1}{2}-y,\frac{1}{2}+x,\frac{1}{2}+z)$	34	Pnn2	35	Cmm
103	P4cc	(x, y, z) (\bar{x}, \bar{y}, z)	$(x, \bar{y}, \frac{1}{2}+z)$ $(\bar{x}, y, \frac{1}{2}+z)$		(y, \bar{x}, z) (\bar{y}, x, z)	27	Pec2	37	Ccc2
104	P4nc	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z)$	$(\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}+z)$ $(\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}+z)$	(y, \bar{x}, z) (\bar{y}, x, z)	34	Pnn2	37	Ccc2
105	P42mc	(x, y, z) (\bar{x}, \hat{y}, z)	(x, \bar{y}, z) (\bar{x}, y, z)	(y , x , $\frac{1}{2}+z$) (\bar{y} , \bar{x} , $\frac{1}{2}+z$)	$(y, \bar{x}, \frac{1}{2}+z)$ $(\bar{y}, x, \frac{1}{2}+z)$	25	Pmm2	37	Cec2
106	P4 ₂ bc	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+x, \frac{1}{2}-y, z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, z)$	$(\frac{1}{2}+y, \frac{1}{2}+x, \frac{1}{2}+z)$ $(\frac{1}{2}-y, \frac{1}{2}-x, \frac{1}{2}+z)$	$(y, \bar{x}, \frac{1}{2}+z)$ $(\bar{y}, x, \frac{1}{2}+z)$	32	Pba2	37	Cce2
107	I4mm	(x, y, z) (\bar{x}, \bar{y}, z)	$ \begin{array}{cccc} (& x, & \bar{y}, & z) \\ (& \bar{x}, & y, & z) \end{array} $	(y, x, z) (\bar{y}, \bar{x}, z)	(y, \bar{x}, z) (\bar{y}, x, z)	44	Imm2	42	Fmm
108	I4cm	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+x, \frac{1}{2}-y, z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, z)$	$(\frac{1}{2}+y, \frac{1}{2}+x, z)$ $(\frac{1}{2}-y, \frac{1}{2}-x, z)$	(y, \bar{x}, z) (\bar{y}, x, z)	45	Iba2	42	Fmm
109	I4 ₁ md	(x, y, z) (\bar{x}, \bar{y}, z)	$ \begin{array}{ccccc} (& x, & \bar{y}, & z) \\ (& \bar{x}, & y, & z) \end{array} $	$(y, \frac{1}{2} + x, \frac{1}{4} + z)$ $(\bar{y}, \frac{1}{2} - x, \frac{1}{4} + z)$	$(y, \frac{1}{2} - x, \frac{1}{4} + z)$ $(\bar{y}, \frac{1}{2} + x, \frac{1}{4} + z)$	44	Imm2	43	Fdd2
110	I41cd	(x, y, z) (\bar{x}, \bar{y}, z)	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$(y, \frac{1}{2} + x, \frac{3}{4} + z)$ $(\bar{y}, \frac{1}{2} - x, \frac{3}{4} + z)$	$(y, \frac{1}{2} - x, \frac{1}{4} + z)$ $(\bar{y}, \frac{1}{2} + x, \frac{1}{4} + z)$	45	Iba2	43	Fdd2
No.	Symbol	1st Subset	2d Subset	1st Subset	2d Subset			No.	Symbo
unstrain order 8	group of ned crystal, per lattice point			ed to axes of unstrained cry that [1 $\overline{1}0$], [110], $z \mid \mid X, Y$,				straine order 4	group of d crystal, per lattice oint

Table 13. Reduction from 422 to 222

orystal, o	of unstrained order 8 per e point		If stressed so that x, y, z Coordinates referred to			crystal,	p of strained order 4 per e point		
No.	Symbol		1st Subset		2d Subset	No.	Symbol		
89	P422	(x, y, z) (\bar{x}, \bar{y}, z)	(x, \bar{y}, \bar{z}) (\bar{x}, y, \bar{z})	(y, x, \bar{z}) $(\bar{y}, \bar{x}, \bar{z})$	(y, \bar{x}, z) (\bar{y}, x, z)	16	P222	21	C222
90	P42 ₁ 2	$ \begin{array}{ccc} (x,y, & z) \\ (\bar{x},\bar{y}, & z) \end{array} $	$(\frac{1}{2}+x,\frac{1}{2}-y, \bar{z})$ $(\frac{1}{2}-x,\frac{1}{2}+y, \bar{z})$	(y, x, \bar{z}) $(\bar{y}, \bar{x}, \bar{z})$	$(\frac{1}{2}+y, \frac{1}{2}-x, z)$ $(\frac{1}{2}-y, \frac{1}{2}+x, z)$	18	$P2_{1}2_{1}2$	21	C222
91	P4 ₁ 22	(x, y, z) $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	$ \begin{array}{cccc} (& x, & \bar{y}, \frac{1}{2} - z) \\ (& \bar{x}, & y, & \bar{z}) \end{array} $	$(y, x, \sqrt[3]{4} - z)$ $(\bar{y}, \bar{x}, \sqrt[1]{4} - z)$	$(y, \bar{x}, \frac{3}{4} + z)$ $(\bar{y}, x, \frac{1}{4} + z)$	17	P222 ₁	20	C222 ₁
92	P4 ₁ 2 ₁ 2	(x, y, z) $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	$(\frac{1}{2}+x, \frac{1}{2}-y, \frac{3}{4}-z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{4}-z)$	(y, x, \bar{z}) $(\bar{y}, \bar{x}, \frac{1}{2} - z)$	$(\frac{1}{2}+y, \frac{1}{2}-x, \frac{3}{4}+z)$ $(\frac{1}{2}-y, \frac{1}{2}+x, \frac{1}{4}+z)$	19	$P2_{1}2_{1}2_{1}$	20	C2221
93	$P4_{2}22$	(x, y, z) (\bar{x}, \bar{y}, z)	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$(y, x, \frac{1}{2} - z)$ $(\bar{y}, \bar{x}, \frac{1}{2} - z)$	$(y, \bar{x}, \frac{1}{2} + z)$ $(\bar{y}, x, \frac{1}{2} + z)$	16	P222	21	C222
94	$P4_{2}2_{1}2$	(x, y, z) (\bar{x}, \bar{y}, z)	$(\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}-z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z)$		$(\frac{1}{2}+y, \frac{1}{2}-x, \frac{1}{2}+z)$ $(\frac{1}{2}-y, \frac{1}{2}+x, \frac{1}{2}+z)$	18	$P2_{1}2_{1}2$	21	C222
95	$P4_{3}22$	(x, y, z) $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	$ \begin{array}{cccc} (& x, & \bar{y}, \frac{1}{2} - z) \\ (& \bar{x}, & y, & \bar{z}) \end{array} $	$(y, x, \frac{1}{4} - z)$ $(\bar{y}, \bar{x}, \frac{3}{4} - z)$	(y , \bar{x} , $\frac{1}{4}+z$) (\bar{y} , x , $\frac{3}{4}+z$)	17	P2221	20	C222 ₁
96	$P4_{3}2_{1}2$	$ \begin{array}{ccc} (x,y,&z) \\ (\bar{x},\bar{y},\frac{1}{2}+z) \end{array} $	$(\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{4}-z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{3}{4}-z)$		$(\frac{1}{2}+y, \frac{1}{2}-x, \frac{1}{4}+z)$ $(\frac{1}{2}-y, \frac{1}{2}+x, \frac{3}{4}+z)$	19	$P2_12_12_1$	20	C222 ₁
97	I422	(x, y, z) (\bar{x}, \bar{y}, z)	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$			23	I222	22	F222
98	I4 ₁ 22	(x, y, z) (\bar{x}, \bar{y}, z)	$(x, \frac{1}{2} - y, \frac{1}{4} - z)$ $(\bar{x}, \frac{1}{2} + y, \frac{1}{4} - z)$		($y, \frac{1}{2} - x, \frac{1}{4} + z$) ($\bar{y}, \frac{1}{2} + x, \frac{1}{4} + z$)	24	$I2_{1}2_{1}2_{1}$	22	F222
No.	Symbol	1st Subset	2d Subset	1st Subset	2d Subset			No.	Symbol
crystal,	o of unstrained order 8 per ce point		Coordinates referred to		and crystal X, Y, Z, any permutation	1			p of straine order 4 per e point

Table 14. Reduction from 6 to 2

	group of ed crystal,	If st	ressed so that $z \parallel X$,	Y, or Z	straine	group of d crystal,
	per lattice pint	Coordinates	order 2 per lattic point			
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbo
168	P6	(x, y, z)	$(\bar{y}, x-y, z)$	$(y-x, \bar{x}, z)$	3	P2
100	10	(\bar{x}, \bar{y}, z)	(y, y-x, z)	(x-y,x,z)		1
169	De	(x, y, z)	$(\bar{y}, x-y, \frac{1}{3}+z)$	$(y-x, \bar{x}, \frac{2}{3}+z)$	4	P31
109	$P6_1$	$(\bar{x}, \bar{y}, \frac{1}{2} + z)$	(y, y-x, 5/6+z)	$(x-y, x, \frac{1}{2}6+z)$	4	Fa
450	To a	(x, y, z)	$(\bar{y}, x-y, \frac{2}{3}+z)$	$(y-x, \bar{x}, \frac{1}{3}+z)$	4	Do
170	P65	$(\bar{x},\bar{y},\frac{1}{2}+z)$	$(y, y-x, \frac{1}{2}6+z)$	(x-y, x, 56+z)	4	P2 ₁
	To a	(x, y, z)	$(\bar{y}, x-y, \frac{2}{3}+z)$	$(y-x, \bar{x}, \frac{1}{3}+z)$		Do
171	$P6_2$	(\bar{x}, \bar{y}, z)	$(y, y-x, \frac{2}{3}+z)$	$(x-y, x, \frac{1}{3}+z)$	3	P2
180	Da l	(x, y, z)	$(\bar{y}, x-y, \frac{1}{3}+z)$	$(y-x, \bar{x}, \frac{2}{3}+z)$	3	P2
172	P6 ₄	(\bar{x}, \bar{y}, z)	$(y, y-x, \frac{1}{3}+z)$	$(x-y, x, \frac{2}{3}+z)$	3	PZ
150	Da	(x, y, z)	$(\bar{y}, x-y, z)$	$(y-x, \bar{x}, z)$		De
173	$P6_3$	$(\bar{x}, \bar{y}, \frac{1}{2} + z)$	$(y, y-x, \frac{1}{2}+z)$	$(x-y, x, \frac{1}{2}+z)$	4	$P2_1$

Table 15. Reduction from $\overline{6}$ to m

	group of ed crystal,	If stresse	Space group of strained crystal,			
	per lattice pint		nates referred to nstrained cryst			er lattice int
No.	Symbol	1st Subset	2d Subset	&d Subset	No.	Symbol
174	₽6	(x, y, z) (x, y, \bar{z})	$(\bar{y}, x-y, z)$ $(\bar{y}, x-y, \bar{z})$	$(y-x,\bar{x},z)$ $(y-x,\bar{x},\bar{z})$	6	Pm

Table 16. Reduction from 3m to m

unstrair order 6	group of ned crystal, per lattice point	If stressed so that	Coordinat	es referred to axes of uns	strained crystal	strained order 2 j	group of 1 crystal, per lattice pint
No.	Symbol		1st Subset	2d Subset	3d Subset	No.	Symbol
156	P3m1	[110] $\parallel X, Y, \text{ or } Z$	(x, y, z) (\hat{y}, \bar{x}, z)	$(y-x, \bar{x}, \qquad z)$ $(x, x-y, \qquad z)$	$(\bar{y}, x-y, z) $ $(y-x, y, z)$	8	Cm
157	P31m	$110 \parallel X, Y, \text{ or } Z$	(x, y, z) (y, x, z)	$(y-x, \bar{x}, \qquad z)$ $(\bar{x}, y-x, \qquad z)$		8	Cm
158	P3c1	[110] X, Y, or Z	(x, y, z) $(\bar{y}, \bar{x}, \frac{1}{2} + z)$	$(y-x, \bar{x}, z)$ $(x, x-y, \frac{1}{2}+z)$	$(\tilde{y}, x-y, z)$ $(y-x, y, \frac{1}{2}+z)$	9	Cc
159	P31c	$1\overline{10} \parallel X, Y, \text{ or } Z$	(x, y, z) $(y, x, \frac{1}{2} + z)$	$(y-x, \bar{x}, z)$ $(\bar{x}, y-x, \frac{1}{2}+z)$	$(\bar{y}, x-y, z)$ $(x-y, \bar{y}, \frac{1}{2}+z)$	9	Сс
160	R3m hex. axes	[110] X, Y, or Z	(x, y, z) (\bar{y}, \bar{x}, z)	$(y-x, \bar{x}, \qquad z)$ $(x, x-y, \qquad z)$	$ \begin{array}{cccc} & (\tilde{y}, x - y, & z) \\ & (y - x, & y, & z) \end{array} $	8	Cm
161	R3c hex. axes	[110] X, Y, or Z	(x, y, z) $(\hat{y}, \bar{x}, \frac{1}{2} + z)$	$(y-x, \bar{x}, z)$ $(x,x-y, \frac{1}{2}+z)$	$(\hat{y}, x-y, z)$ $(y-x, y, \frac{1}{2}+z)$	9	Сс

Table 17. Reduction from 32 to 2

unstraine order 6 p	group of ed crystal, per lattice int	If stressed so that	Coordinate	es referred to axes of un	nstrained crystal	strained order 2 p	group of crystal, per lattice int
No.	Symbol		1st Subset	2d Subset	3d Subset	No.	Symbol
149	P312	$110 \parallel X, Y, \text{ or } Z$	(x, y, z) $(\bar{y}, \bar{x}, \bar{z})$	$(\bar{y}, x-y, z) $ $(y-x, y, \bar{z})$		5	C2
150	P321	[110] X, Y, or Z	(x, y, z) (y, x, \bar{z})	$(\bar{y}, x-y, z) $ $(x-y, \bar{y}, \bar{z})$	$(y-x, \bar{x}, \qquad z)$ $(\bar{x}, y-x, \bar{z})$	5	C2
151	P3 ₁ 12	$[1\overline{1}0] \parallel X, Y, \text{ or } Z$	(x, y, z) $(\bar{y}, \bar{x}, \frac{2}{3} - z)$	$(\bar{y}, x-y, \frac{1}{3}+z)$ $(y-x, y, \frac{1}{3}-z)$	$(y-x, \bar{x}, \frac{2}{3}+z)$ $(x, x-y, \bar{z})$	5	C2
152	P3 ₁ 21	[110] X, Y, or Z	(x, y, z) (y, x, \bar{z})	$(x-y, \frac{1}{3}+z)$ $(x-y, \frac{\bar{y}}{3}, \frac{2}{3}-z)$	$(y-x, \bar{x}, \frac{2}{3}+z) \\ (\bar{x}, y-x, \frac{1}{3}-z)$	5	C2
153	P3 ₂ 12	$1\overline{10} \parallel X, Y, \text{ or } Z$	(x, y, z) $(\bar{y}, \bar{x}, \frac{1}{3} - z)$	$(\bar{y}, x-y, \frac{2}{3}+z)$ $(y-x, y, \frac{2}{3}-z)$	$(y-x, \bar{x}, \frac{1}{3}+z)$ $(x, x-y, \bar{z})$	5	C2
154	P3 ₂ 21	[110] X, Y, or Z	(x, y, z) (y, x, \bar{z})	$(\bar{y}, x-y, \frac{2}{3}+z)$ $(x-y, \bar{y}, \frac{1}{3}-z)$	$(y-x, \bar{x}, \frac{1}{3}+z)$ $(\bar{x}, y-x, \frac{2}{3}-z)$	5	C2
155 hex.	R32 axes	[110] X, Y, or Z	(x, y, z) (y, x, \bar{z})	$ \begin{array}{cccc} (&\bar{y},x-y,&&z)\\ (x-y,&\bar{y},&\bar{z}) \end{array} $		5	C2

Table 18. Reduction from 4 to 2

	group of ed crystal,	If stress	sed so t	hat z]	Y, Y,	or Z		group of crystal,	
order 4	per lattice oint			referred ned crys		s of	order 2 per lattice point		
No.	Symbol	1st Sul	oset	2d	Subse	t	No.	Symbo	
75	P4	(x, y,	<i>z</i>)	$(\bar{y},$	<i>x</i> ,	z)	3	P2	
10	11	$(\bar{x}, \bar{y},$	z)	(y,	\bar{x} ,	z)		12	
76	P4 ₁	(x, y,	z)	$(\bar{y},$	x, 1/4	(+z)	4	P21	
70	1 41	$(\bar{x}, \bar{y}, \frac{1}{2})$	(+z)	(y,	\bar{x} , $\frac{3}{4}$	(+z)	-1	1 21	
77	P42	(x, y,	z)	$(\bar{y},$	x, ½	(+z)	3	P2	
"	1 42	$(\bar{x}, \bar{y},$	<i>z</i>)	(y,	\bar{x} , $\frac{1}{2}$	+z)		12	
78	P43	(x, y,	z)	$(\bar{y},$	x, 3/4	(+z)	4	P21	
10	1 13	$(\bar{x}, \bar{y}, \frac{1}{2})$	(±+z)	(y,	\bar{x} , $\frac{1}{4}$	(+z)	1	1 -1	
79	14	(x, y,	z)	$(\bar{y},$	x,	z)	5	C2	
79	14	$(\bar{x}, \bar{y},$	<i>z</i>)	(y,	\bar{x} ,	<i>z</i>)	9	C2	
80	T4	(x, y,	z)	$(\bar{y}, \frac{1}{2})$	$+x, \frac{1}{4}$	(+z)	5	C2	
80	I4 ₁	$(\bar{x}, \bar{y},$	<i>z</i>)	$(y, \frac{1}{2})$	$-x, \frac{1}{4}$	(+z)	5	C2	

Table 19. Reduction from $\frac{1}{4}$ to 2

	group of	If stressed so tha	at $z \parallel X$, Y , or Z	Space g	
order 4 p	ed crystal, er lattice int	Coordinates ref unstraine		strained order 2 p po	
No.	Symbol	1st Subset	2d Subset	No.	Symbol
81	₽4	(x, y, z) (\bar{x}, \hat{y}, z)	(\bar{y}, x, \bar{z}) (y, \bar{x}, \bar{z})	3	P2
82	14	(x, y, z) (\bar{x}, \bar{y}, z)	(\bar{y}, x, \bar{z}) (y, \bar{x}, \bar{z})	5	C2

Table 20. Reduction from mm2 to m

Space unstrain	group of ed crystal,	If stressed so the	at $x \parallel X$, Y , or Z	Space	group of d crystal,	If stressed so the	at $y \parallel X$, Y , or Z	Space	group of ed crystal,
order 4	per lattice oint	Coordinates referred to a	xes of unstrained crystal	order 2	per lattice oint	Coordinates referred to a	exes of unstrained crystal	order 2	per lattice point
No.	Symbol	1st Subset	2d Subset	No.	Symbol	1st Subset	2d Subset	No.	Symbol
25	Pmm2	((x, \bar{y}, z) (\bar{x}, \bar{y}, z)	6	Pm	(x, y, z) (x, \bar{y}, z)	(\bar{x}, y, z) (\bar{x}, \bar{y}, z)	6	Pm
26	$Pmc2_1$	($(x, \bar{y}, \frac{1}{2} + z)$ $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	6	Pm	(x, y, z) $(x, \bar{y}, \frac{1}{2} + z)$	(\bar{x}, y, z) $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	7	Pe
27	Pec2	(x, y, z) $(\bar{x}, y, \frac{1}{2} + z)$	(x,	7	Pc	(x, y, z) $(x, \bar{y}, \frac{1}{2} + z)$	$(\bar{x}, y, \frac{1}{2} + z)$ (\bar{x}, \bar{y}, z)	7	Рс
28	Pma2		$\begin{array}{ccc} (\frac{1}{2}+x, & \bar{y}, & z) \\ (& \bar{x}, & \bar{y}, & z) \end{array}$	6	Pm	(x, y, z) $(\frac{1}{2}+x, \bar{y}, z)$	$(\frac{1}{2}-x, y, z)$ (\bar{x}, \bar{y}, z)	7	Рс
29	Pca2 ₁	(x, y, z) $(\frac{1}{2}-x, y, \frac{1}{2}+z)$	$(\frac{1}{2}+x, \bar{y}, z)$ $(\bar{x}, \bar{y}, \frac{1}{2}+z)$	7	Pe		$(\frac{1}{2}-x, \qquad y, \frac{1}{2}+z)$ $(\qquad \bar{x}, \qquad \bar{y}, \frac{1}{2}+z)$	7	Рс
30	Pnc2	(x, y, z) $(\bar{x}, \frac{1}{2} + y, \frac{1}{2} + z)$	$(x, \frac{1}{2} - y, \frac{1}{2} + z)$ (\bar{x}, \bar{y}, z)	7	Pe	(x, y, z) $(x, \frac{1}{2} - y, \frac{1}{2} + z)$	$(\bar{x}, \frac{1}{2} + y, \frac{1}{2} + z)$ (\bar{x}, \bar{y}, z)	7	Ре
31	Pmn2 ₁	(x, y, z) (\bar{x}, y, z)	$(\frac{1}{2}+x, \bar{y}, \frac{1}{2}+z)$ $(\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z)$	6	Pm	(x, y, z) $(\frac{1}{2}+x, \bar{y}, \frac{1}{2}+z)$	(\bar{x}, y, z) $(\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z)$	7	Pe
32	Pba2	(x, y, z) $(\frac{1}{2}-x, \frac{1}{2}+y, z)$	$(\frac{1}{2}+x,\frac{1}{2}-y, \qquad z)$ $(\bar{x}, \bar{y}, \qquad z)$	7	Pe	(x, y, z) $(\frac{1}{2}+x, \frac{1}{2}-y, z)$	$(\frac{1}{2}-x,\frac{1}{2}+y, \qquad z)$ $(\bar{x}, \bar{y}, \qquad z)$	7	Рс
33	Pna2 ₁	(x, y, z) $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z)$	$(\frac{1}{2}+x, \frac{1}{2}-y, z)$ $(\bar{x}, \bar{y}, \frac{1}{2}+z)$	7	Pe	(x, y, z) $(\frac{1}{2}+x, \frac{1}{2}-y, z)$	$(\frac{1}{2}-x,\frac{1}{2}+y,\frac{1}{2}+z)$ $(\bar{x}, \bar{y},\frac{1}{2}+z)$	7	Рс
34	Pnn2	(x, y, z) $(\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}+z)$	$(\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z)$ (\bar{x}, \bar{y}, z)	7	Pe	(x, y, z) $(\frac{1}{2}+x, \frac{1}{2}-y, \frac{1}{2}+z)$	$(\frac{1}{2}-x,\frac{1}{2}+y,\frac{1}{2}+z)$ (\bar{x}, \bar{y}, z)	7	Pe
35	Cmm2	($ \begin{array}{cccccccccccccccccccccccccccccccccccc$	8	Cm	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	(\bar{x}, y, z) (\bar{x}, \bar{y}, z)	8	Cm
36	Cmc2 ₁	(x, y, z) (\bar{x}, y, z)	(x,	8	Cm	(x, y, z) $(x, \bar{y}, \frac{1}{2} + z)$	(\bar{x}, y, z) $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	9	Се
37	Ccc2	(x, y, z) $(\bar{x}, y, \frac{1}{2} + z)$	(x,	9	Сс	(x, y, z) $(x, \bar{y}, \frac{1}{2} + z)$	$(\bar{x}, y, \frac{1}{2} + z)$ (\bar{x}, \bar{y}, z)	9	Сс
38	Amm2	(x, y, z) (\bar{x}, y, z)	(x, \bar{y}, z) (\bar{x}, \bar{y}, z)	6	Pm	(x, y, z) (x, \bar{y}, z)	(8	Cm
39	Abm2	(x, y, z) $(\bar{x}, \frac{1}{2} + y, z)$	$(x, \frac{1}{2} - y, z)$ (\bar{x}, \bar{y}, z)	7	Pe	(x, y, z) $(x, \frac{1}{2} - y, z)$	$(\bar{x}, \frac{1}{2} + y, z)$ (\bar{x}, \bar{y}, z)	8	Cm
40	Ama2	(x, y, z) $(\frac{1}{2}-x, y, z)$	$(\frac{1}{2}+x, \bar{y}, z)$ (\bar{x}, \bar{y}, z)	6	Pm	(x, y, z) $(\frac{1}{2}+x, \bar{y}, z)$	$(\frac{1}{2}-x, y, z)$ (\bar{x}, \bar{y}, z)	9	Сс
41	Aba2	(x, y, z) $(\frac{1}{2}-x, \frac{1}{2}+y, z)$	$(\frac{1}{2}+x,\frac{1}{2}-y, \qquad z)$ $(\bar{x}, \bar{y}, \qquad z)$	7	Pc	(x, y, z) $(\frac{1}{2}+x, \frac{1}{2}-y, z)$	$(\frac{1}{2}-x,\frac{1}{2}+y, \qquad z)$ $(\bar{x}, \bar{y}, \qquad z)$	9	Сс
42	Fmm2	(x, y, z) (\bar{x}, y, z)	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	8	Cm	(x, y, z) (x, \bar{y}, z)	(\bar{x}, y, z) (\bar{x}, \bar{y}, z)	8	Cm
43	Fdd2	(x, y, z) $(\frac{1}{4}-x, \frac{1}{4}+y, \frac{1}{4}+z)$	$(\frac{1}{4}+x, \frac{1}{4}-y, \frac{1}{4}+z)$ (\bar{x}, \bar{y}, z)	9	Сс	(x, y, z) (1/4+x, 1/4-y, 1/4+z)	$(\frac{1}{4}-x,\frac{1}{4}+y,\frac{1}{4}+z)$ (\bar{x}, \bar{y}, z)	9	Сс
44	Imm2	($(\qquad x, \qquad \bar{y}, \qquad z)$ $(\qquad \bar{x}, \qquad \bar{y}, \qquad z)$	8	Cm	(x, y, z) (x, \bar{y}, z)	(\bar{x}, y, z) (\bar{x}, \bar{y}, z)	8	Cm
45	Iba2	(x, y, z) $(\bar{x}, y, \frac{1}{2} + z)$	(9	Се	($ \begin{array}{cccc} (& \bar{x}, & y, \frac{1}{2} + z) \\ (& \bar{x}, & \bar{y}, & z) \end{array} $	9	Сс
46	Ima2	(x, y, z) $(\frac{1}{2}-x, y, z)$	$(\frac{1}{2}+x, \bar{y}, z)$ (\bar{x}, \bar{y}, z)	8	Cm		$(\frac{1}{2}-x, y, z)$ (\bar{x}, \bar{y}, z)	9	Сс

Table 21. Reduction from mm2 to 2

	group of ed crystal,	If stressed so	that $z \parallel X$. Y , or Z	Space g	group of
order 4	per lattice oint		referred to axes of ined crystal	order 2 p	l crystal, per lattice pint
No.	Symbol	1st Subset	2d Subset	No.	Symbo
25	Pmm 2	(x, y, z) (\bar{x}, \bar{y}, z)	(x, \bar{y}, z) (\bar{x}, y, z)	3	P2
26	Pmc2 ₁	(x, y, z) $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	$(x, \bar{y}, \frac{1}{2} + z)$ (\bar{x}, y, z)	4	P2 ₁
27	Pec2	$ \begin{array}{ccc} (& x, y, & z) \\ (& \bar{x}, \bar{y}, & z) \end{array} $	$(x, \bar{y}, \frac{1}{2}+z)$ $(\bar{x}, y, \frac{1}{2}+z)$	3	P2
28	Pma2	$ \begin{array}{cccc} (& x, y, & z) \\ (& \bar{x}, \bar{y}, & z) \end{array} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	3	P2
29	Pca2 ₁	$ \begin{array}{ccc} (& x, y, & z) \\ \hline (& x, y, & z) \\ (& \bar{x}, \bar{y}, \frac{1}{2} + z) \end{array} $	$(\frac{1}{2}+x, \bar{y}, z)$ $(\frac{1}{2}-x, y, \frac{1}{2}+z)$	4	P2 ₁
30	Pne2	(x, y, z)	$(x, \frac{1}{2} - y, \frac{1}{2} + z)$	3	P2
31	Pmn2 ₁	$ \begin{array}{ccc} (& \bar{x}, \bar{y}, & z) \\ \hline (& x, y, & z) \end{array} $	$(\bar{x}, \frac{1}{2} + y, \frac{1}{2} + z)$ $(\frac{1}{2} + x, \bar{y}, \frac{1}{2} + z)$	4	P2 ₁
32	Pba2	$\frac{(\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z)}{(x, y, z)}$	$ \frac{(\bar{x}, y, z)}{(\frac{1}{2} + x, \frac{1}{2} - y, z)} $	3	P2
33	Pna2 ₁	$ \begin{array}{ccc} (& \bar{x}, \bar{y}, & z) \\ \hline (& x, y, & z) \end{array} $	$(\frac{1}{2}-x, \frac{1}{2}+y, z)$ $(\frac{1}{2}+x, \frac{1}{2}-y, z)$	4	P2 ₁
34	Pnn2	$ \begin{array}{c c} (& \bar{x}, \bar{y}, \frac{1}{2} + z) \\ \hline (& x, y, z) \end{array} $	$\frac{(\frac{1}{2}-x,\frac{1}{2}+y,\frac{1}{2}+z)}{(\frac{1}{2}+x,\frac{1}{2}-y,\frac{1}{2}+z)}$	-	P2
	-	$ \begin{array}{c cc} (& \bar{x}, \bar{y}, & z) \\ \hline (& x, y, & z) \end{array} $	$\frac{(\frac{1}{2}-x,\frac{1}{2}+y,\frac{1}{2}+z)}{(x, \bar{y}, z)}$	- 3	
35	Cmm2	$ \begin{array}{c ccc} (& \bar{x}, \bar{y}, & z) \\ \hline (& x, y, & z) \end{array} $	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	- 3	P2
36	Cmc2 ₁	$(\bar{x}, \bar{y}, \frac{1}{2} + z)$	$($ $\bar{x},$ $y,$ $z)$	4	P2 ₁
37	Ccc2	(x, y, z) (\bar{x}, \bar{y}, z)	$(x, \tilde{y}, \frac{1}{2}+z)$ $(x, y, \frac{1}{2}+z)$	3	P2
38	Amm2	(x, y, z) (\bar{x}, \bar{y}, z)	(x,	5	C2
39	Abm2	$ \begin{array}{cccc} (& x,y, & z) \\ (& \bar{x},\bar{y}, & z) \end{array} $	$(x, \frac{1}{2} - y, z)$ $(\bar{x}, \frac{1}{2} + y, z)$	5	C2
40	Ama2	$ \begin{array}{cccc} (& x,y, & z) \\ (& \bar{x},\bar{y}, & z) \end{array} $	$(\frac{1}{2}+x, \bar{y}, z)$ $(\frac{1}{2}-x, y, z)$	5	C2
41	Aba2	$ \begin{array}{ccc} (& x, y, & z) \\ (& \bar{x}, \bar{y}, & z) \end{array} $	$(\frac{1}{2}+x, \frac{1}{2}-y, z)$ $(\frac{1}{2}-x, \frac{1}{2}+y, z)$	5	C2
42	Fmm2	$ \begin{array}{c cccc} (& x, y, & z) \\ \hline (& \bar{x}, \bar{y}, & z) \end{array} $	(x, \overline{y}, z) (\bar{x}, y, z)	5	C2
43	Fdd2	(x,y,z)	$(\frac{1}{4}+x,\frac{1}{4}-y,\frac{1}{4}+z)$	5	C2
44	Imm2	$ \begin{array}{ccc} (& \bar{x}, \bar{y}, & z) \\ \hline (& x, y, & z) \end{array} $	$\frac{(\frac{1}{4} - x, \frac{1}{4} + y, \frac{1}{4} + z)}{(x, \bar{y}, z)}$	5	C2
45	Iba2	$ \begin{array}{ccc} (& \bar{x}, \bar{y}, & z) \\ \hline (& x, y, & z) \end{array} $	(5	C2
	-	$ \begin{array}{ccc} (& \bar{x}, \bar{y}, & z) \\ \hline (& x, y, & z) \end{array} $	$(\tilde{x}, y, \frac{1}{2} + z)$ $(\frac{1}{2} + x, \tilde{y}, z)$	-	
46	Ima2	$($ $\bar{x}, \bar{y},$ $z)$	$(\frac{1}{2}-x, y, z)$	5	C2

Table 22. Reduction from 222 to 2

of un	e group strained	If stressed so	that $x \parallel X$, Y , or Z	of st	e group crained	If stressed so the	at $y \parallel X$, Y , or Z	of s	e group trained	If stressed so	that $z \parallel X$, Y , or Z	of st	e group trained
4 per	al, order r lattice point		s referred to axes of ained crystal	2 per	al, order lattice oint		ferred to axes of ed crystal	2 pe	al, order r lattice point		es referred to axes of crained crystal	2 per	al, order r lattice oint
No.	Symbol	1st Subset	2d Subset	No.	Symbol	1st Subset	2d Subset	No.	Symbol	1st Subset	2d Subset	No.	Symbol
16	P222	(x, y, z) (x, \bar{y}, \bar{z})	$($ $\bar{x},$ $y,$ $\bar{z})$ $($ $\bar{x},$ $\bar{y},$ $z)$	3	P2	((x,	3	P2	(x, y, z) (\bar{x}, \bar{y}, z)	(3	P2
17	P222 ₁	(x, y, z) (x, \bar{y}, \bar{z})	(\bar{x} , $y, \frac{1}{2}-z$) (\bar{x} , \bar{y} , $\frac{1}{2}+z$)	3	P2	(x, y, z) $(\bar{x}, y, \frac{1}{2} - z)$	(x, \bar{y}, \bar{z}) $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	3	P2	(x, y, z) $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	(x, \bar{y}, \bar{z}) $(\bar{x}, y, \frac{1}{2} - z)$	4	P2 ₁
18	P2 ₁ 2 ₁ 2	(x, y, z) $(\frac{1}{2}+x, \frac{1}{2}-y, \bar{z})$	$(\frac{1}{2}-x,\frac{1}{2}+y, \qquad \bar{z})$ $(\bar{x}, \bar{y}, \qquad z)$	4	P2 ₁	(x, y, z) $(\frac{1}{2}-x, \frac{1}{2}+y, \bar{z})$	$ \begin{array}{ccc} \hline (\frac{1}{2}+x,\frac{1}{2}-y, & \bar{z}) \\ (& \bar{x}, & \bar{y}, & z) \end{array} $	4	P2 ₁	$ \begin{array}{cccc} (& x,y, & z) \\ (& \bar{x},\bar{y}, & z) \end{array} $	$(\frac{1}{2}+x,\frac{1}{2}-y, \bar{z})$ $(\frac{1}{2}-x,\frac{1}{2}+y, \bar{z})$	3	P2
19	P2 ₁ 2 ₁ 2 ₁	(x, y, z) $(\frac{1}{2}+x, \frac{1}{2}-y, \bar{z})$	$(\bar{x}, \frac{1}{2} + y, \frac{1}{2} - z) $ $(\frac{1}{2} - x, \bar{y}, \frac{1}{2} + z)$	4	P2 ₁	(x, y, z) $(\bar{x}, \frac{1}{2} + y, \frac{1}{2} - z)$	$(\frac{1}{2}+x,\frac{1}{2}-y, \bar{z})$ $(\frac{1}{2}-x, \bar{y},\frac{1}{2}+z)$	4	P2 ₁	(x, y, z) $(\frac{1}{2} - x, \bar{y}, \frac{1}{2} + z)$	$(\frac{1}{2}+x,\frac{1}{2}-y, \bar{z})$ $(\bar{x},\frac{1}{2}+y,\frac{1}{2}-z)$	4	P2 ₁
20	C222 ₁	(x, y, z) (x, \bar{y}, \bar{z})	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	5	C2	(x, y, z) $(\bar{x}, y, \frac{1}{2} - z)$	(x, \bar{y}, \bar{z}) $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	5	C2	(x, y, z) $(\bar{x}, \bar{y}, \frac{1}{2} + z)$	(x, \bar{y}, \bar{z}) $(\bar{x}, y, \frac{1}{2} - z)$	4	P2 ₁
21	C222	(x, y, z) (x, \bar{y}, \bar{z})	(\bar{x}, y, \bar{z}) (\bar{x}, \bar{y}, z)	5	C2	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		5	C2	$ \begin{array}{cccc} (& x,y, & z) \\ (& \bar{x},\bar{y}, & z) \end{array} $	(x, \bar{y}, \bar{z}) (\bar{x}, y, \bar{z})	3	P2
22	F222	(x, y, z) (x, \bar{y}, \bar{z})	(5	C2	(x, y, z) (\bar{x}, y, \bar{z})		5	C2	(x, y, z) (\bar{x}, \bar{y}, z)	(x, \bar{y}, \bar{z}) (\bar{x}, y, \bar{z})	5	C2
23	I222	((\bar{x}, y, \bar{z}) (\bar{x}, \bar{y}, z)	5	C2	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		5	C2	(x, y, z) (\bar{x}, \bar{y}, z)	(x, \bar{y}, \bar{z}) (\bar{x}, y, \bar{z})	5	C2
24	I2 ₁ 2 ₁ 2 ₁	(x, y, z) $(\frac{1}{2}+x, \frac{1}{2}-y, \bar{z})$	$(\bar{x}, \frac{1}{2} + y, \frac{1}{2} - z) $ $(\frac{1}{2} - x, \bar{y}, \frac{1}{2} + z)$	5	C2	(x, y, z) $(\bar{x}, \frac{1}{2} + y, \frac{1}{2} + z)$	$(\frac{1}{2}+x,\frac{1}{2}-y, \bar{z})$ $(\frac{1}{2}-x, \bar{y},\frac{1}{2}+z)$	5	C2	(x, y, z) $(\frac{1}{2}-x, \bar{y}, \frac{1}{2}+z)$	$(\frac{1}{2}+x,\frac{1}{2}-y, \bar{z})$ $(\bar{x},\frac{1}{2}+y,\frac{1}{2}-z)$	5	C2

Table 23. Reduction from 3 to 1

unstrai	e group of ned crystal,	No	specialization o	f stress	strained	roup of crystal,
	per lattice point	Coordinate	es referred to axe crystal	s of unstrained		er lattice int
No.	Symbol	1st Subset	2d Subset	3d Subset	No.	Symbol
143	P3	(x, y, z)	$(\bar{y}, x-y, z)$	$(y-x,\bar{x}, z)$	1	P1
144	$P3_1$	(x, y, z)	$(\bar{y}, x-y, \frac{1}{3}+z)$	$(y-x, \bar{x}, \frac{2}{3}+z)$	1	P1
145	$P3_2$	(x, y, z)	$(\bar{y}, x-y, \frac{2}{3}+z)$	$(y-x, \bar{x}, \frac{1}{3}+z)$	1	P1
146	R3 hex. axes	(x, y, z)	$(\bar{y}, x-y, z)$	$(y-x,\bar{x}, z)$	1	P1

Table 24. Reduction from m to 1

Space	group of	No specializ	zation of stress	Space group of		
order 2 p	ed crystal, per lattice pint	of unstra	referred to axes ined crystal algorithms.	order 1 p	crystal, per lattice int	
No.	Symbol	1st Subset	2d Subset	No.	Symbol	
6	Pm	(x, y, z)	(x, \bar{y}, z)	1	P1	
7	Pe	(x, y, z)	$(x, \hat{y}, \frac{1}{2} + z)$	1	Pi	
8	Cm	(x, y, z)	(x, \bar{y}, z)	1	P1	
9	Сс	(x, y, z)	$(x, \bar{y}, \frac{1}{2} + z)$	1	P1	

Table 25. Reduction from 2 to 1

Space group of unstrained crystal, order 2 per lattice point		No specializ	zation of stress	Space group of strained crystal, order 1 per lattice point		
		of unstra	referred to axes ined crystal 2 y)			
No.	Symbol	1st Subset	2d Subset	No.	Symbol	
3	P2	(x, y, z)	(\bar{x}, y, \bar{z})	1	P1	
4	$P2_1$	(x, y, z)	$(\bar{x}x, \frac{1}{2}+y, \bar{z})$	1	P1	
5	C2	(x, y, z)	(\bar{x}, y, \bar{z})	1	P1	

 ${\bf T_{ABLE}}\ 26.\ Most\ general\ stress\ consistent\ with\ a\ reduction\ of\ a\ noncentrosymmetric\ point\ group\ to\ any\ one\ of\ its\ stress-induced\ subgroups$

	Subgroup										
Initial group					→Compou	·Compound					
	Self subgroup	4 2m	422	3m	32	3	mm2	222	m	2	1
43m	X=Y=Z	$X=Y$; z $\parallel Z$		$X=Y$; [111] $\parallel Z$			[1 $\overline{10}$], [110], $z \parallel X$, Y , Z , any permutation	$x, y, z \parallel X, Y, Z,$ any permutation	$\begin{bmatrix} 110] \parallel \\ X, Y, \text{ or } \\ Z \end{bmatrix}$	$X, \stackrel{x \parallel}{Y}, \text{ or } Z$	Any
432	X=Y=Z		$X=Y; z \parallel Z$		X = Y; [111] Z			x, y, z or $[1\overline{1}0]$, $[110], z \parallel X, Y, Z$, any permutation		$\begin{array}{c} x \text{ or } [110] \\ \parallel X, Y, \\ \text{ or } Z \end{array}$	Any
23	X=Y=Z					$X=Y;$ [111] $\parallel Z$		$x, y, z \parallel X, Y, Z,$ any permutation		$X, \stackrel{\parallel}{Y}, \text{ or } Z$	Any

	Subgroup								
Initial group	Cicle	mr-9			9	1			
	Self subgroup	mm2	222	m	2	1			
6m2 (hex. axes)	$X=Y, z \parallel Z$	[1 $\bar{1}$ 0], [110], $z \parallel X$, Y , Z , any permutation		$z \parallel X, Y, \text{ or } Z \text{ or } F \text{ or } 6\text{m}2: \ [110]; \text{ for } 62\text{m}: [1\overline{10}] \parallel X, Y, \text{ or } Z$	$\begin{bmatrix} \text{For } \overline{6} \text{ m2:} \\ [1\overline{1}0]; \text{ for } \\ \overline{6}2\text{m:} & [110] \\ \parallel X, Y, \text{ or } \end{bmatrix}$	Any			
6mm (hex. axes)	$X = Y,$ $z \parallel Z$	$z, y \parallel \text{any two}$ of X, Y, Z		[110] or $[1\overline{1}0]$ $\parallel X, Y, \text{ or } Z$	X, Y, or Z	Any			
622 (hex. axes)	$X=Y,$ $z\parallel Z$		$z, y \parallel \text{any two}$ of X, Y, Z		$ \begin{array}{c} [110], [1\overline{1}0], \text{ or } z \\ \parallel X, Y, \text{ or } Z \end{array} $	Any			
(hex. axes)	$X=Y,$ $z \parallel Z$				X, Y, or Z	Any			
(hex. axes)	$X=Y,$ $z \parallel Z$			$X, \stackrel{z}{Y}, \text{ or } Z$		Any			
42m	$X=Y,$ $z\parallel Z$	For $\overline{4}2m$: [1 $\overline{1}0$], [110], 2; for $\overline{4}m$ 2: $x, y, z \parallel X$, Y, Z , any permutation	For $\overline{4}$ 2m: x, y, z ; for $\overline{4}$ m2: $[1\overline{10}]$, $[110]$, $z \parallel$ X, Y, Z, any permutation	$ \left[\begin{array}{c} \text{For $\overline{4}2\text{m}:$} \\ [110]; \text{ for } \\ \underline{4\text{m}2:} \ x \parallel \\ X, Y, \text{ or } \\ Z \end{array} \right] $	$\begin{bmatrix} \text{For } \overline{4}2\text{m:} \\ x; \text{ for } \overline{4}\text{m2:} \\ [110] \parallel X, \\ Y, \text{ or } Z \\ \text{or } z \parallel X, \\ Y, \text{ or } Z \end{bmatrix}$	Any			
4mm	$X=Y, z \parallel Z$	x, y, z or $[1\overline{1}0]$, $[110]$, $z \parallel X$, Y , Z , any permutation		$x \text{ or } [110]$ $\parallel X, Y, \text{ or } Z$	$Z \parallel X, Y, \text{ or } Z$	Any			
422	$X=Y, z \parallel Z$		$x, y, z \text{ or } [1\overline{1}0],$ $[110], z \parallel X,$ Y, Z, any permutation		x , [110], or z $\parallel X$, Y , or Z	Any			
4	$X=Y,$ $z \parallel Z$				X, Y, or Z	Any			
- 4	$X=Y,$ $z \parallel Z$				Z, Y, or Z	Any			
3m (hex. axes)	$X=Y, \ z\parallel Z$			For 3m1: [110]; for 31m: [1 $\overline{10}$] $\parallel X, Y$, or Z		Any			
(hex. axes)	$X=Y,$ $z\parallel Z$				$ \begin{bmatrix} \text{For 321:} \\ [110]; \text{for} \\ 312: [1\overline{10}] \\ \parallel X, Y, \text{ or} \\ Z \end{bmatrix} $	Any			
(hex. axes)				-		Any			
mm2	$x, y, z \parallel X, Y, Z,$ any perm.			$\parallel X, Y, \text{ or } Z$	$z \parallel X, Y, \text{ or } Z$	Any			
222	$x, y, z \mid X, Y, Z,$ any perm.					Any			
m(<u></u> y)	$X, \stackrel{y}{\underset{Z}{\parallel}}$, or					Any			
2(y)	$X, \stackrel{y}{\stackrel{\parallel}{X}}, \text{ or } Z$					Any			
1	Any					X			

Large square brackets indicate same subgroup in equivalent setting.